

# Lorentz Transformation as a Galilei Transformation with Physical Length and Time Contractions

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The Lorentz transformations between the space-time coordinates of a point in two inertial frames with arbitrary relative velocity, are reformulated as Galilei transformations with length and time contractions, by introducing the ether rest frame (in which light signals propagate isotropically with the vacuum speed of light). The generalized Galilei transformations for the (longitudinal) space coordinates ( $x_{1,2}$ ) and the time variables ( $t_{1,2}$ ) of a point in two inertial frames  $\Sigma_{1,2}$  are not only of analogous structure, but have remarkable symmetry properties, too. The appearing length and time contractions are absolute effects in the sense of Lorentz-Fitzgerald, i.e., a rod has its largest length and a clock its fastest rate when at rest in the ether frame  $\Sigma_0$ . Thus, an analytical reformulation and a physical interpretation of the Lorentz transformations within Galilean relativity physics is achieved.

## Introduction

The covariance properties of Maxwell's equations were first investigated by Voigt [1], Lorentz [2], Poincaré [3], and Einstein [4]. The emerging "Lorentz transformations" (rotations in the complex 4-dimensional  $r$ , ict-space) [3] have been accepted as the physically relevant transformation group [5]. Thus, it has been postulated that all laws of physics have to be covariant under the space-time transformations of Lorentz ( $L$ ) [6]. The requirement of  $L$ -covariance is the cause of the infamous difficulties of modern physics (point particles with infinite self-energy and self-acceleration, infinite zero-point electromagnetic energy density of the vacuum, violation of causality principle, relativistic paradoxes, etc).

Bateman and Cunningham found later (1909) that Maxwell's equations are also covariant under nonlinear *conformal* transformations [7, 8]. Recently, Fushchich and Nikitin discovered several other transformations of Maxwell's equations by means of group analysis [9, 10]. Winterberg showed that the introduction of a minimum length leads to nonlinear  $L$ -transformations, which remove the singularities of quantum electrodynamics on a physical basis [11]. According to a conjecture by Post [5, 12–14], the  $L$  and Galilei ( $G$ ) transformations are special cases of a more

general group of nonlinear space-time coordinate substitutions. Wilhelm found that the transformation functions, which leave the electromagnetic wave equation form-invariant, are determined by coupled linear partial differential equations which have an infinite number of linear and nonlinear solutions [15] (the  $L$ -transformation representing a simple linear solution). In this connection, the "non-uniqueness" of the covariance of the Maxwell equations is obvious.

A problem in the intuitive comprehension of the  $L$ -transformation is the underlying relativity principle [4], according to which one and the same light signal propagates isotropically in free space with the same speed  $c$  not only in the rest frame of the source but also in any one of the infinite number of possible inertial frames. On the other hand, the Galilean relativity principle implies that electromagnetic waves propagate (i) isotropically with velocity  $c$  only in the preferred inertial frame in which the electromagnetic ether is at rest and (ii) anisotropically in all inertial frames which move relative to the ether ("ether" designates the real substratum which carries the electromagnetic waves [2–3, 11, 17–10, 20, 22–29]).

Thus, the physical principles underlying the  $L$ - and  $G$ -transformations appear to be different to the point of contradicting each other. For this reason, the theoretical findings to be presented are remarkable, since they show that the  $L$ -transformations can be brought into the form of generalized  $G$ -transformations with length and time contractions. The proposed Galilei transformations are deduced on the hypothesis of a

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preferred frame of reference, the ether rest-frame, in which light propagates isotropically [16], and are, therefore, not physically equivalent to the Lorentz transformations. The length and time contractions are not independent effects, since a clock can be visualized as a light signal reflected to and fro in the ether between two mirrors held apart by a rod [17–19]. The general theory of such light-clocks has been given by Winterberg [20].

Our considerations are in line with numerous publications on absolute space-time concepts, which are experimentally supported by the discovery of the 2.7 K microwave background of the universe [21]. Absolute space-time concepts were originally conceived by Galilei, Newton, Faraday, Maxwell, Hertz, Heaviside, Lorentz, Poincaré, and Abraham [22]. Einstein foresaw the significance of the electromagnetic ether for the further development of physics in this way [23]: “According to the general theory of relativity space without ether is unthinkable; for in such space there not only would be no propagation of light, but also no possibility of existence for standards of space and time (measuring rods and clocks), nor therefore any space-time in a physical sense.”

The consideration of the electromagnetic ether resulted in the *physical* resolution of several “theoretical difficulties” of present physics. E.g., a physical theory of length and time contractions (originally postulated to be independent [4]) as interdependent ether effects [17–20], a quantum electrodynamics with a minimum length which eliminates the unphysical (i) point particles and (ii) etherless vacuum [11], a Galilei covariant formulation of electromagnetic theory [24] and Schroedinger’s wave mechanics [25], and a vector theory of gravitation in ether [26] which agrees with all known linear and nonlinear experiments on gravitation. The latter theory is very promising since it is free from causality violating solutions and the fatal Penrose singularities (which induced many theorists to discard the general theory of relativity).

Herein, we consider the space-time coordinates  $(x_1 y_1 z_1 t_1)$  and  $(x_2 y_2 z_2 t_2)$  of a point P in the inertial frames  $\Sigma_1$  and  $\Sigma_2$  with arbitrary relative velocity,  $|u_2 - u_1| < c$ . These space-time coordinates are connected through the *L*-transformations [4]. By relating the space-time coordinates (i) of P in  $\Sigma_1$  and (ii) P in  $\Sigma_2$  to the coordinates  $(x_0 y_0 z_0 t_0)$  of P in the ether rest frame  $\Sigma_0$ , the *L*-transformations are reformulated as generalized *G*-transformations. These differ from the primitive *G*-transformations through (i) the length and

time contractions for objects (“rods” and “clocks”) moving relative to the ether, and (ii) the time-separation effect  $\sim u_{12} x_0 / c^2$ , resulting from the relative motion  $u_{12} = u_2 - u_1$  of the inertial frames  $\Sigma_{1,2}$ . Thus, a new formulation of the *L*-transformations within the frame of Galilean relativity physics is obtained.

### Interrelation of Symmetric *L*- and *G*-Transformations

In Galilean physics, the preferred or ether rest frame is defined as that inertial frame, in which electromagnetic waves propagate isotropically with the “vacuum” speed of light  $c = (\mu_0 \epsilon_0)^{-1/2} \cong 3 \times 10^8$  m/s [15]. Other properties of the electromagnetic ether are its electric permittivity,  $\epsilon_0 \cong 10^{-9}/36\pi$  As/Vm, magnetic permeability,  $\mu_0 = 4\pi \times 10^{-7}$  Vs/Am, and its electromagnetic wave resistance  $Z_0 = (\mu_0/\epsilon_0)^{1/2} \cong 120\pi$  V/A [17, 24, 27].

We consider first the mathematically simple case for inertial frames  $\Sigma_1$  and  $\Sigma_2$  (space and time coordinates:  $x_i y_i z_i t_i$ ,  $i = 1, 2$ ), which move *symmetrically* with the velocities  $\mathbf{u}_1 = -\mathbf{u}$  and  $\mathbf{u}_2 = +\mathbf{u}$  parallel to the  $\pm x_0$ -axes of the ether rest frame (space and time coordinates:  $x_0 y_0 z_0 t_0$ ) where  $|\mathbf{u}| \leq c$  is arbitrary (Figure 1). An arbitrary point P with the coordinates  $(x_0 y_0 z_0 t_0)$  in  $\Sigma_0$  has then the coordinates  $(x_1 y_1 z_1 t_1)$  in  $\Sigma_1$  and  $(x_2 y_2 z_2 t_2)$  in  $\Sigma_2$ . The coordinate axes of  $\Sigma_0$ ,  $\Sigma_{1,2}$  are assumed to coincide for  $t_1 = t_2 = t_0 = 0$  (Figure 1). In  $\Sigma_0$  (ether velocity  $\mathbf{w} = \mathbf{0}$ ) light propagates isotropically (*G*-time  $t_0$ ) [15]. By Galilean relativity, light propagates anisotropically in  $\Sigma_{1,2}$  (ether velocity  $\mathbf{w}_{1,2} = \mp \mathbf{u}$ ) [16].

According to the *L*-transformation, the coordinates  $x_1 \dots t_1$  and  $x_2 \dots t_2$  of P in the moving inertial frames  $\Sigma_1$  and  $\Sigma_2$  are related to the coordinates  $x_0 \dots t_0$  of P in the ether rest frame  $\Sigma_0$  by [4, 16] (Figure 1):

$$x_0 = \gamma_0 (x_1 - u t_1), \quad (1)$$

$$t_0 = \gamma_0 (t_1 - u c^{-2} x_1), \quad (2)$$

and

$$x_0 = \gamma_0 (x_2 + u t_2), \quad (3)$$

$$t_0 = \gamma_0 (t_2 + u c^{-2} x_2), \quad (4)$$

where

$$\gamma_0 = (1 - u^2/c^2)^{-1/2}, \quad u_2 = -u_1 = u, \quad (5)$$

and  $y_i = y_0$ ,  $z_i = z_0$ ,  $i = 1, 2$ . Equations (1) and (3) have a physical explanation in terms of the Lorentz-Fitzgerald contraction [22]. The length of a rod ex-

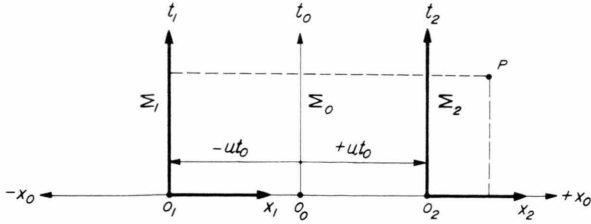


Fig. 1. Inertial frames  $\Sigma_1(x_1 t_1)$  and  $\Sigma_2(x_2 t_2)$  moving symmetrically in ether rest frame  $\Sigma_0(x_0 t_0)$ .

tending from  $0_0$  to  $x_0$  along the  $x_0$ -axis is (i)  $x_0$  in  $\Sigma_0$ , (ii)  $x_1 - u t_1 = x_0/\gamma_0$  in  $\Sigma_1$ , and (iii)  $x_2 + u t_2 = x_0/\gamma_0$  in  $\Sigma_2$  (Figure 1). Thus, the rod has the “largest” length  $x_0$  in  $\Sigma_0$  ( $\mathbf{w} = \mathbf{0}$ ) and the “contracted” length  $x_0/\gamma_0$  for observers in  $\Sigma_{1,2}$  ( $\mathbf{w}_{1,2} = \mp \mathbf{u}$ ) since  $\gamma_0^{-1} < 1$ . By the inverse of (2) and (4), an event of duration  $\Delta t_0$  at  $(x_0 y_0 z_0)$  of  $\Sigma_0$  is observed to have a dilated duration  $\Delta t_{1,2} = \gamma_0 \Delta t_0$  in  $\Sigma_{1,2}$ , i.e.,  $\Delta t_0 = \Delta t_{1,2}/\gamma_0$  is contracted in comparison to  $\Delta t_{1,2}$ .

From the  $L$ -transformation (1), (3) and (2), (4), we find by elimination of  $x_0$  respectively  $t_{1,2}$  the following simple and transparent relations between the coordinates  $x_{1,2}$  of the point P in  $\Sigma_{1,2}$  ( $y_1 = y_2, z_1 = z_2$ ):

$$x_1/\gamma_0 = x_2/\gamma_0 + u_{12} t_0, \quad (6)$$

$$x_2/\gamma_0 = x_1/\gamma_0 - u_{12} t_0, \quad (7)$$

where

$$u_{12} = u_2 - u_1 = 2u \quad (8)$$

in view of the symmetric motion of  $\Sigma_{1,2}$  in  $\Sigma_0$  (Figure 1). Equations (6) and (7) are the generalized Galilei transformations with length contraction, between the coordinates  $x_1$  and  $x_2$  of P in  $\Sigma_{1,2}$ . They reduce to the ordinary  $G$ -transformations for  $\gamma_0 \rightarrow 1$  ( $u/c \rightarrow 0$ ).

The  $G$ -transformations have a clear physical interpretation in the ether rest frame  $\Sigma_0$ .  $x_1/\gamma_0$  and  $x_2/\gamma_0$  are the contracted coordinate distances  $x_1$  and  $x_2$  which an observer in  $\Sigma_0$  measures. Furthermore,  $u_{12}$  is the relative velocity of  $\Sigma_2$  and  $\Sigma_1$  in  $\Sigma_0$ , and  $t_0$  is the time observed in  $\Sigma_0$ .

In a similar way, the  $G$ -transformations between the times  $t_{1,2}$  are found. Elimination of  $t_0$  from (2) and (4) yields

$$t_1 - t_2 = u c^{-2} (x_1 + x_2), \quad (9)$$

where

$$x_1 + x_2 = 2\gamma_0 x_0 \quad (10)$$

by (1) and (3). Equations (9) and (10) combine to the generalized  $G$ -transformations with time contraction

between the times  $t_1$  and  $t_2$  of the point P in the moving inertial frames  $\Sigma_{1,2}$ :

$$t_1/\gamma_0 = t_2/\gamma_0 + u_{12} c^{-2} \cdot x_0, \quad (11)$$

$$t_2/\gamma_0 = t_1/\gamma_0 - u_{12} c^{-2} \cdot x_0, \quad (12)$$

where  $u_{12}$  is defined in (8). They reduce to the primitive  $G$ -transformations  $t_1 = t_2$  in the nonrelativistic limit  $u/c \rightarrow 0$  ( $\gamma_0 \rightarrow 1$ ).

The  $G$ -transformations (11)–(12) are readily interpreted in the ether rest frame  $\Sigma_0$ . Whereas  $t_{1,2}$  are observed in  $\Sigma_{1,2}$ ,  $t_1/\gamma_0$  and  $t_2/\gamma_0$  are the corresponding contracted time periods observable in  $\Sigma_0$ . That the interrelation of the times  $t_1$  and  $t_2$  depends on the position  $x_0$  of P in  $\Sigma_0$  is an interesting “relativistic” effect. The time-difference  $t_1 - t_2$  is small, except for  $|u| \rightarrow c$  ( $\gamma_0 \rightarrow \infty$ ) and large coordinate values  $|x_0| \sim c t_{1,2}$  of P in  $\Sigma_0$ .

It should be noted that the times  $t_1$  and  $t_2$  in the  $G$ -transformations (11)–(12) refer to planes  $x_{1,2}$  that are separated by (i) a distance  $x_1$  from  $O_1$  (according to an observer at  $O_1$ ) and (ii) a distance  $x_2$  from  $O_2$  (according to an observer at  $O_2$ ). By (6)–(7), the planes  $x_1/\gamma_0$  and  $x_2/\gamma_0$  are a distance  $u_{12} t_0$  apart in  $\Sigma_0$ ,  $(x_1 - x_2)/\gamma_0 = u_{12} t_0$ . Thus, in  $\Sigma_0$  the time-difference between the planes  $x_{1,2}$  is  $(t_1 - t_2)/\gamma_0 = u_{12} t_0/c = u_{12} x_0/c^2$ , in accord with (11)–(12). Here,  $t_0 = x_0/c$  for a clock at  $x_0 \neq 0$  which is synchronized with the clock at  $x_0 = 0$  ( $O_1 = O_2 = O_0$  for  $t_1 = t_2 = t_0 = 0$ , Figure 1).

Thus, we have demonstrated that the coordinates  $x_{1,2}$  and times  $t_{1,2}$  of a point P in the inertial frames  $\Sigma_{1,2}$  (with arbitrary relative velocity  $u_{12}$ ) are interrelated through generalized  $G$ -transformations with length and time contraction effects, in the case of symmetrical translations  $u_{1,2} = \mp u$  of  $\Sigma_{1,2}$  in the ether rest frame  $\Sigma_0$  (Figure 1). For comparison, the corresponding  $L$ -transformations are stated ( $y_1 = y_2, z_1 = z_2$ ) [4, 17]:

$$x_1 = \bar{\gamma}(x_2 + \bar{v}_{12} t_2), \quad t_1 = \bar{\gamma}(t_2 + \bar{v}_{12} c^{-2} x_2), \quad (13)$$

$$x_2 = \bar{\gamma}(x_1 - \bar{v}_{12} t_1), \quad t_2 = \bar{\gamma}(t_1 - \bar{v}_{12} c^{-2} x_1), \quad (14)$$

where

$$\bar{\gamma} = (1 - \bar{v}_{12}^2/c^2)^{-1/2} \quad (15)$$

and

$$\begin{aligned} \bar{v}_{12} &= (u_2 - u_1)/(1 - u_1 u_2/c^2) \\ &= 2u/(1 + u^2/c^2). \end{aligned} \quad (16)$$

The nonlinear Einsteinian relative velocity  $\bar{v}_{12}$  is the velocity of  $\Sigma_2$  as seen from  $\Sigma_1$  [4]. Equations (13)–(14) follow from (1)–(4) by elimination of  $x_0$  and  $t_0$  (in

accordance with the group property of the  $L$ -transformation). The complicated structure of (13)–(16) and their disapproval to physical interpretation are obvious.

By the derivations given, the generalized  $G$ -transformations (6)–(7), (11)–(12) are mathematically but not physically equivalent reformulations of the  $L$ -transformations (13)–(14). The proposed transformations provide a lucid understanding of the interrelation of the space and time coordinates of an arbitrary point in the two inertial frames  $\Sigma_{1,2}$  within Galilean relativity physics when  $\Sigma_1$  and  $\Sigma_2$  move symmetrically in the ether frame  $\Sigma_0$  (Figure 1). It is seen that  $x_{1,2}$  and  $t_{1,2}$  transform in an analogous manner, as is evident from the similarity of (i) the length and time contractions  $\sim \gamma_0^{-1}$  [(6)–(7); (11)–(12)] and (ii) the spatial coordinate separation  $\pm u_{12} t_0$  [(6)–(7)] and the time variable separation  $\pm u_{12} x_0/c^2$  [(11)–(12)].

### Interrelation of Arbitrary $L$ - and $G$ -transformations

In general, the inertial frames  $\Sigma_{1,2}$  will move with arbitrary, non-symmetric velocities  $u_1 \neq u_2$ ,  $|u_{1,2}| < c$ , relative to the ether rest frame  $\Sigma_0$ . We prove now that generalized  $G$ -transformations with length and time contractions result also from the  $L$ -transformations for the space and time coordinates of a point  $P$  in two arbitrary inertial frames  $\Sigma_{1,2}$  (Figure 2).

Figure 2 depicts two inertial frames  $\Sigma_{1,2}$  (with space and time coordinates  $x_i y_i z_i t_i$ ,  $i = 1, 2$ ), which move with arbitrary (non-symmetric) velocities  $u_1 \neq u_2$  parallel to the  $x_0$ -axis of the ether rest frame  $\Sigma_0$  (the vector components  $u_{1,2}$  may have opposite signs or the same signs). We consider an arbitrary point  $P$ , which has the coordinates  $(x_1 y_1 z_1 t_1)$  in  $\Sigma_1$ ,  $(x_2 y_2 z_2 t_2)$  in  $\Sigma_2$ , and  $(x_0 y_0 z_0 t_0)$  in  $\Sigma_0$ . As usual, the coordinate axes of  $\Sigma_0$  and  $\Sigma_{1,2}$  are assumed to coincide for  $t_1 = t_2 = t_0 = 0$  (Figure 2).

According to the  $L$ -transformation, the coordinates  $x_1 \dots t_1$  and  $x_2 \dots t_2$  of  $P$  in the moving inertial frames  $\Sigma_{1,2}$  are related to the coordinates  $x_0 \dots t_0$  of  $P$  in the ether rest frame  $\Sigma_0$  by [4, 17] (Figure 2):

$$x_0 = \gamma_1(x_1 + u_1 t_1), \quad (17)$$

$$t_0 = \gamma_1(t_1 + u_1 c^{-2} x_1), \quad (18)$$

and

$$x_0 = \gamma_2(x_2 + u_2 t_2), \quad (19)$$

$$t_0 = \gamma_2(t_2 + u_2 c^{-2} x_2), \quad (20)$$

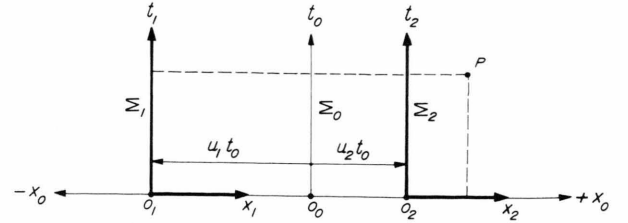


Fig. 2. Inertial frames  $\Sigma_1(x_1 t_1)$  and  $\Sigma_2(x_2 t_2)$  moving asymmetrically in ether rest frame  $\Sigma_0(x_0 t_0)$ .

where

$$\gamma_1 = (1 - u_1^2/c^2)^{-1/2}, \quad \gamma_2 = (1 - u_2^2/c^2)^{-1/2} \quad (21)$$

and  $y_i = y_0$ ,  $z_i = z_0$ ,  $i = 1, 2$ . Elimination of  $x_0$  from (17) and (19) gives

$$\gamma_1(x_1 + u_1 t_1) = \gamma_2(x_2 + u_2 t_2), \quad (22)$$

where

$$t_1 = t_0/\gamma_1 - u_1 c^{-2} x_1, \quad (23)$$

$$t_2 = t_0/\gamma_2 - u_2 c^{-2} x_2, \quad (24)$$

by (18) and (20). Substitution of  $t_1$  and  $t_2$  into (22) yields the fundamental  $G$ -transformations between the coordinates  $x_1$  and  $x_2$  of the point  $P$  in the moving ( $u_1 \neq u_2$ ) inertial frames  $\Sigma_{1,2}$ :

$$x_1/\gamma_1 = x_2/\gamma_2 + u_{12} t_0, \quad (25)$$

$$x_2/\gamma_2 = x_1/\gamma_1 - u_{12} t_0, \quad (26)$$

where

$$u_{12} = u_2 - u_1 \geq 0 \quad (27)$$

and  $y_1 = y_2$ ,  $z_1 = z_2$ .  $u_{12}$  is the relative velocity of  $\Sigma_1$  and  $\Sigma_2$  measured in  $\Sigma_0$ . These generalized  $G$ -transformations contain the length contraction effects through the Lorentz factors  $\gamma_{1,2}$ .

For the physical interpretation of (25) and (26) it is noted that (i)  $t_0$  is the  $G$ -time observed in  $\Sigma_0$  and (ii)  $x_1/\gamma_1$  and  $x_2/\gamma_2$  are the contracted ( $\gamma_{1,2} > 1$ ) coordinates of  $P$  in  $\Sigma_{1,2}$  which an observer in  $\Sigma_0$  measures.  $u_{12}$  is the relative  $G$ -velocity of  $\Sigma_1$  and  $\Sigma_2$  in  $\Sigma_0$ . Accordingly, (25) and (26) represent generalized  $G$ -transformations between the coordinates  $x_1$  and  $x_2$  of  $\Sigma_{1,2}$ , “projected” into the ether rest frame  $\Sigma_0$ .

For the derivation of the  $G$ -transformations between the times  $t_1$  and  $t_2$  of  $\Sigma_{1,2}$ ,  $\gamma_1 t_1$  and  $\gamma_2 t_2$  are eliminated from the  $L$ -transformations (18) and (20) and subtracted,

$$\gamma_1 t_1 - \gamma_2 t_2 = c^2(\gamma_2 u_2 x_2 - \gamma_1 u_1 x_1), \quad (28)$$



where

$$\begin{aligned} & \gamma_2 u_2 x_2 - \gamma_1 u_1 x_1 \\ &= (u_2 - u_1) x_0 - \gamma_2 u_2^2 t_2 + \gamma_1 u_1^2 t_1 \end{aligned} \quad (29)$$

by (17) and (19). Substitution of (29) into (28) results in the fundamental Galilei transformations with time contraction for the times  $t_1$  and  $t_2$  of the point P in the moving ( $u_1 \neq u_2$ ) inertial frames  $\Sigma_{1,2}$ :

$$t_1/\gamma_1 = t_2/\gamma_2 + u_{12} c^{-2} x_0, \quad (30)$$

$$t_2/\gamma_2 = t_1/\gamma_1 - u_{12} c^{-2} x_0. \quad (31)$$

The physical meaning of (30) and (31) is obvious in the ether rest frame  $\Sigma_0$ . The contracted ( $\gamma_{1,2} > 1$ ) times  $t_1/\gamma_1$  and  $t_2/\gamma_2$  of  $t_{1,2}$  of  $\Sigma_{1,2}$  are observable in  $\Sigma_0$ . The relative  $G$ -velocity  $u_{12}$  of  $\Sigma_1$  and  $\Sigma_2$  is measurable in  $\Sigma_0$ , and  $x_0$  is the coordinate of P in  $\Sigma_0$ . The interrelation between the times  $t_1$  and  $t_2$  depends on the position  $x_0$  of P in  $\Sigma_0$ , representing a “relativistic” time-separation effect. The time difference  $t_1 - t_2$  is small, except for  $|u_{12}| \rightarrow c (\gamma_{1,2} \rightarrow \infty)$  and large  $|x_0| \sim c t_{1,2}$  coordinates of P in  $\Sigma_0$ .

For comparison, the known  $L$ -transformations between the coordinates  $x_1 \dots t_1$  and  $x_2 \dots t_2$  of P in the (moving) inertial frames  $\Sigma_{1,2}$  are given ( $y_1 = y_2$ ,  $z_1 = z_2$ ) [4, 17]:

$$x_1 = \gamma(x_2 + v_{12} t_2), \quad t_1 = \gamma(t_2 + v_{12} c^{-2} x_2), \quad (32)$$

$$x_2 = \gamma(x_1 - v_{12} t_1), \quad t_2 = \gamma(t_1 - v_{12} c^{-2} x_1), \quad (33)$$

where

$$\gamma = (1 - v_{12}^2/c^2)^{-1/2}, \quad (34)$$

$$v_{12} = (u_2 - u_1)/(1 - u_1 u_2/c^2). \quad (35)$$

Equation (35) defines the relativistic relative velocity of  $\Sigma_2$  and  $\Sigma_1$  [4]. Equations (32) and (33) are directly derivable from (17)–(20) by elimination of  $x_0$  and  $t_0$ . In contrast to the generalized  $G$ -transformations (25)–(26) and (30)–(31), the Lorentz transformations (32)–(35) are less symmetric and have a relative velocity  $v_{12}$  which is *nonlinear* in  $u_1$  and  $u_2$ , (35).

Thus, we have demonstrated that (i) the coordinates  $x_1$  and  $x_2$  and (ii) times  $t_1$  and  $t_2$  of a point P in the moving ( $u_1 \neq u_2$ ) inertial frames  $\Sigma_1$  and  $\Sigma_2$  are interrelated by the generalized  $G$ -transformations (25)–(26) and (30)–(31) with length and time contractions. These transformations hold for inertial frames  $\Sigma_{1,2}$  with (i) arbitrary relative velocities  $|u_{1,2}| < c$  and (ii) arbitrary (non-symmetric) velocities  $u_{1,2}$  relative to the ether rest frame  $\Sigma_0$ ,  $|u_1| \neq |u_2|$ .

The generalized  $G$ -transformations (25)–(26) and (30)–(31) are mathematically but not physically equiv-

alent to the  $L$ -transformations (32)–(33), by the derivations given. The generalized  $G$ -transformations have essential advantages, (i) they are analytically simple and (ii) they are understandable by the natural concepts of Galilean relativity physics. The spatial coordinate separation in (25)–(26), as seen from the ether frame  $\Sigma_0$ ,

$$x_1/\gamma_1 - x_2/\gamma_2 = u_{12} t_0 \quad (36)$$

is a readily understandable effect caused by the relative motion  $u_{12} = u_2 - u_1$  of  $\Sigma_{1,2}$  (Figure 2). The associated separation of the time variables is in the ether frame  $\Sigma_0$  (with velocity of light  $c$ )

$$t_1/\gamma_1 - t_2/\gamma_2 = u_{12} t_0/c = u_{12} x_0/c^2 \quad (37)$$

in accord with (30)–(31). If a clock at a point  $(0, y_0, z_0)$  of the  $x_0 = 0$  plane emits a light signal at time  $t_0 = 0$ , then a *synchronized* clock at a point  $(x_0, y_0, z_0)$  of the  $x_0 \neq 0$  plane receives it at time  $t_0 = x_0/c$  sec. Accordingly, (37) represents a relativistic effect ( $t_0 \rightarrow 0$ ,  $c \rightarrow \infty$ ), which is due to the simple fact that a light signal requires a nonvanishing time  $t_0 = x_0/c$  to travel the distance  $x_0$  in the ether rest frame.

## Conclusions

We have shown that the  $L$ -transformations can be reformulated in the form of generalized  $G$ -transformations with length and time contractions by considering the physical effects of the electromagnetic ether and the ether frame on relativistic phenomena. The generalized  $G$ -transformations reduced to the primitive  $G$ -transformations in the limit of infinite speeds of light propagation:  $x_2 = x_1 - u_{12} t_0$ ,  $t_2 = t_1 = t_0$ ,  $c \rightarrow \infty$ .

The generalized  $G$ -transformations with length and time contractions explain relativistic phenomena in a natural way through the Galilean relativity concepts. These involve, inter alia, anisotropic light propagation in all inertial frames except the ether rest frame. In Galilean physics, length and time contractions are not “relativistic” phenomena but interdependent physical effects in the sense of Lorentz-Fitzgerald [17, 22] (flattening of the Coulomb fields of matter in motion through the ether) [11, 17, 20, 27].

In the ether rest frame  $\Sigma_0$ , a rod has its largest length  $l_0$  and a clock its fastest rate  $v_0$ , when these objects are at rest there. For this reason,  $l_0$  and  $v_0$  have the physical meaning of “proper length” of the rod and

of “proper frequency” of the clock, respectively [24]. In the etherless relativity theory (STR), the proper length and proper frequency are defined as the observables in the frame  $\Sigma$  moving with the rod or clock [4] (no matter whether  $\Sigma$  actually moves relative to  $\Sigma_0$  or not [4]). Thus, the purely kinematic length and time contractions of the STR “observed” for rods and clocks at rest in  $\Sigma_0$  from a moving frame  $\Sigma$  are deceptions created by the real deformations of the measuring devices attached to  $\Sigma$ .

These differences between the kinematic–fictitious (STR) and the real length and time contractions for objects moving relative to the ether ( $\Sigma_0$ ) have to be kept in mind in applying the generalized  $G$ -transformations in (25)–(26) and (30)–(31). These read in vector notation (“ $\parallel$ ” and “ $\perp$ ” to  $\mathbf{u}_{1,2}$ ):

$$\mathbf{r}_2^\parallel/\gamma_2 = \mathbf{r}_1^\parallel/\gamma_1 - \mathbf{u}_{12} t_0, \quad \mathbf{r}_2^\perp = \mathbf{r}_1^\perp, \quad (38)$$

$$t_2/\gamma_2 = t_1/\gamma_1 - \mathbf{u}_{12} \cdot \mathbf{r}_0/c^2. \quad (39)$$

The corresponding transformations for  $\mathbf{r}_1$  and  $t_1$  follow from (38) and (39) by interchange of the subscripts “1” and “2”, where

$$\mathbf{u}_{12} = -\mathbf{u}_{21}, \quad \mathbf{u}_{12} = \mathbf{u}_2 - \mathbf{u}_1, \quad (40)$$

$$\gamma_i = (1 - \mathbf{u}_i^2/c^2)^{-1/2} > 1, \quad \text{for } |\mathbf{u}_i| < c; \quad (41)$$

$$i = 1, 2.$$

In addition to the higher symmetry of (38) and (39), the proposed transformations have a Galilean relative velocity  $\mathbf{u}_{12}$  which is linear in  $\mathbf{u}_1$  and  $\mathbf{u}_2$ , (40). E.g., two systems  $\Sigma_1$  and  $\Sigma_2$  moving with the velocities  $u_{1,2} = \pm 9c/10$  in a reference frame  $\Sigma$  have the relative velocity  $u_{12} = 18c/10$  in  $\Sigma$ . On the other hand, the Einsteinian relative velocity  $v_{12}$ , (35), in the corresponding  $L$ -transformations is nonlinear in  $u_1$  and  $u_2$ , and is not at all defined in the reference frame  $\Sigma$  in

which  $u_{1,2}$  are defined! Rather,  $v_{12}$  is believed to be the velocity of  $\Sigma_1$  as seen from  $\Sigma_2$  [4, 17–19, 22].

Length and time contractions are real dynamic processes resulting from the interaction of matter with the electromagnetic ether (substratum) [2, 3, 16–27]. As a body or clock changes its velocity  $v(t)$  with time  $t$  in a frame of reference  $\Sigma$ , the length  $l(v)$  or the frequency  $\nu(v)$  measured in  $\Sigma$  do not adapt instantaneously to the changing velocity  $v(t)$ , but with a nonvanishing relaxation time,  $\tau > 0$  [20, 28, 29]. For this reason, the generalized  $G$ -transformations are not applicable to non-inertial frames with characteristic times  $t \lesssim \tau$ , but they may be applied (as an approximation) to slowly accelerated non-inertial frames ( $t \gg \tau$ ).

Observation of relaxation processes in length and time contractions would render possible the experimental determination of the preferred frame of reference  $\Sigma_0$  (in which the ether is at rest and light propagates isotropically) at any location of space. Lorentz invariance violating effects in rotating systems (electron in atom, bending waves in rods), in particular under conditions of resonance ( $\omega \sim \tau^{-1}$ ), have recently been analyzed by Winterberg and discussed with regard to ether experiments [20, 28, 29]. This research is very important for settling the question of the electromagnetic ether experimentally, the existence of which is the hypothesis underlying this communication\*.

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